Pressure of a gas

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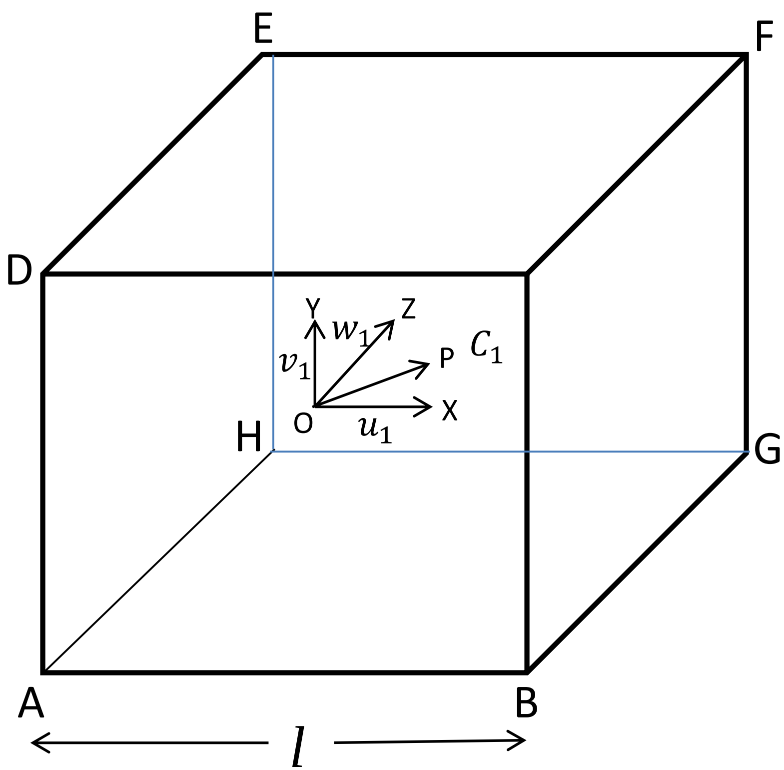

Consider a cubical vessel ABCDEFGH containing gas.

The volume of the vessel = $l^3\ cc$

m – mass of each molecule
n – number of molecules

Consider a molecule P moving in a random direction with velocity C_1.

The velocity components are u_1, v_1 and w_1 along XYZ axes.

Therefore the resultant velocity is

$$C_1{}^2 = u_1{}^2 + v_1{}^2 + w_1{}^2$$

Let the molecule strike on the wall BCFG with velocity u_1 *then the momentum is* mu_1.
The same molecule strike on the opposite wall ADEH then the momentum is $-mu_1$.

The change in momentum due to impact

$$mu_1 - (-mu_1) = 2mu_1$$

The time interval between two successive impacts on the wall BCFG is

$$t = \frac{distance\ covered}{velocity\ along\ X\ axis} = \frac{2l}{u_1}$$

$$No.impacts\ per\ second = \frac{1}{t} = \frac{1}{\frac{2l}{u_1}} = \frac{u_1}{2l}$$

$$Change\ in\ momentum\ produced\ in\ one\ second$$

$$due\ to\ the\ impact\ of\ this\ molecule\ = 2mu_1 \times \frac{u_1}{2l} = \frac{m{u_1}^2}{l}$$

The force Fx due to the impact of all the n molecules in one second

$$= \frac{m}{l}[{u_1}^2 + {u_2}^2 + \cdots + {u_n}^2]$$

Force per unit area on the wall BCFG or ADEH is equal to the pressure Px

$$P_X = \frac{m}{l \times l^2}[{u_1}^2 + {u_2}^2 + \cdots + {u_n}^2]$$

Similarly the pressure P_Y on the walls CDEF and ABGH

$$P_Y = \frac{m}{l \times l^2}[{v_1}^2 + {v_2}^2 + \cdots + {v_n}^2]$$

Similarly the pressure P_Z on the walls ABCD and EFGH

$$P_Z = \frac{m}{l \times l^2}[{w_1}^2 + {w_2}^2 + \cdots + {w_n}^2]$$

$$u_1 = \frac{l}{t}$$

$$\frac{u_1}{l} = \frac{1}{t}$$

$$force = \frac{p}{t}$$

$$= p \times \frac{1}{t}$$

$$= mu_1 \times \frac{u_1}{l}$$

$$= \frac{m}{l}{u_1}^2$$

As the pressure of the gas is same in all directions, the mean pressure P is

$$P = \frac{P_X + P_Y + P_Z}{3}$$

$$= \frac{m}{3l^3}\left[\begin{matrix}(u_1{}^2 + v_1{}^2 + w_1{}^2) + (u_2{}^2 + v_2{}^2 + w_2{}^2) + (u_3{}^2 + v_3{}^2 + w_3{}^2) \\ \ldots. + (u_n{}^2 + v_n{}^2 + w_n{}^2)\end{matrix}\right]$$

$$= \frac{m}{3l^3}\left[C_1{}^2 + C_2{}^2 + C_3{}^2 \ldots. + C_n{}^2\right] --- (i)$$

$But\ V = l^3.\ Let\ C\ be\ the\ r.m.s\ velocity\ of\ the\ molecules$

$$C^2 = \frac{C_1{}^2 + C_2{}^2 + C_3{}^2 \ldots. + C_n{}^2}{n}$$

$$nC^2 = C_1{}^2 + C_2{}^2 + C_3{}^2 \ldots. + C_n{}^2$$

Sub in (i)

$$P = \frac{m.nC^2}{3V} = \frac{MC^2}{3V} = \frac{\rho C^2}{3}$$

$$m.n = M$$

$$C^2 = \frac{3P}{\rho}$$

$$r.m.s\ velocity\ of\ the\ molecules\ C = \sqrt{\frac{3P}{\rho}}$$